Power System State Estimation using Synchrophasor Measurements

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Summary

- **Power system dynamics**
- **Data acquisition systems**
  - SCADA
  - PMU
- **Static State Estimation**
  - SCADA based
  - PMU based
  - Hybrid
- **Dynamic State Estimation**
  - Dynamic estimation of the quasi-\textit{static state}
  - Dynamic estimation of the \textit{electromechanical state}
State Estimation

- Find the optimal estimate of a stochastic dynamic system expected state consistent with both observations and the system dynamic model.
Dynamic Models

- **Discrete Linear Time-Invariant Model**

\[ x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \]
\[ z_k = Hx_k + v_k \]
\[ w_k \sim (0, Q_k) \]
\[ v_k \sim (0, R_k) \]

- **Discrete Nonlinear Time-Invariant Model**

\[ x_k = f(x_{k-1}, u_{k-1}, w_{k-1}) \]
\[ z_k = h(x_k) + v_k \]
\[ w_k \sim (0, Q_k) \]
\[ v_k \sim (0, R_k) \]

- \( x_k \): state vector
- \( u_k \): control vector
- \( z_k \): measurement vector
- \( w_k \): system noise vector (represents uncertainty in the dynamic models)
- \( v_k \): measurement error vector

- \( Q_k \): noise covariance matrix
- \( R_k \): measurement error covariance matrix
Slow or *Quasi* Static Dynamics

- The state variables change very little between sampling times
- State variables (**static state**): \( x_k = \begin{bmatrix} V_k & \theta_k \end{bmatrix} \)
- The system dynamics is due to
  - Load variation
  - Generation variation (wind, solar)
  - Slow controllers (LTC, etc.)
- Can be predict by short term load forecasting and power flow models: closed form is impractical
- State estimation application
  - Dynamic model is neglected
  - State estimates at relatively large intervals (minutes)
  - Mathematical Model
    
    \[
    z_k = h(x_k) + v_k \\
    v_k \sim (0, R_k)
    \]
  - Usually only one measurement snapshot (\( Z_k \)) processed at a time
Fast Dynamics

- Electromechanical transients
- Horizons
  - Short term (few seconds): movement of rotors and fast controllers action
  - Long term (several second to minutes): includes prime movers slow dynamics and slow controllers

- State variables: \( x_k = [\delta_k \omega_k \cdots] \)

- Elaborated models available for machines, controllers, etc.

- State estimation application
  - Made possible owing to the availability of synchrophasor measurements
  - Practical application not yet clearly defined
  - Mathematical Model

\[
\begin{align*}
x_k &= f(x_{k-1}, u_{k-1}, w_{k-1}) \\
z_k &= h(x_k) + v_k \\
w_k &\sim (0, Q_k) \\
v_k &\sim (0, R_k)
\end{align*}
\]
Data Acquisition Systems

- **SCADA**
  - Components
    - Man-Machine Interface
    - Computational Systems
    - Remote Terminal Units
    - Sensors
    - Telecommunication System
  - Functions
    - Data acquisition and display
    - Remote actuation
  - Poll Interval: 2-4 seconds
  - No data synchronization: skew

- **PMU Network**
  - PMU: Phasor Measurement Units (frequency, voltage and current positive sequence phasors)
  - Time synchronization
  - PDC Phasor Data Concentrators
    - Substation level
    - Regional level
    - Central PDC
  - LAN and Wans: usually private owned
  - 10 to 60 phasors per second

PMU Data Delay

SCADA Hypothetical Example

- 2 seconds poll interval

![Diagram showing a power system with V_t and \( \delta_t \)]
PMU Hypothetical Example

- **30 samples/sec**

![Graph showing phase angle δₜ over time](image)

<table>
<thead>
<tr>
<th>Phasor Reporting Rate (Hz)</th>
<th>Cut-off Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>30</td>
</tr>
<tr>
<td>30</td>
<td>15</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>15</td>
<td>7.5</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

*Source: Phadke & Thorp book (2008)*
Power System State Estimation

- **Estimation of the Quasi-Static State**
  - Usually referred to as **Static State Estimation**
  - State variables: steady state nodal voltages \((V, \theta)\)
  - Measurements: SCADA + PMU
  - Applications
    - Unit commitment and economic dispatch
    - Voltage and loading monitoring
    - Initial conditions for on-line VSA and DSA

- **Estimation of the Electromechanical State**
  - Truly **Dynamic State Estimation**
  - State Variables: generator/controllers variables
    - \((\delta, \omega, E_{fd}, \ldots)\)
  - Measurements: PMU only
  - Applications:
    - Out-of-step protection
    - Generator model identification
    - Other?
Static State Estimation

- **Conventional power system state estimation**
  - Processing one snapshot of SCADA data at a time
  - Rate of processing: minutes (30 – 90 seconds)
  - Large research effort in the last ~40 years
  - Introduced by F. Schweppe (1970s)
  - Summarized in books: Monticelli, Abur & Expósito
  - Measurement gross errors (bad data) and topology errors are still not completely solved problems (practical importance?)
  - Largely used by industry but with only reasonable performance

- **Dynamic state estimation of the static state**
  - Attempts to use past information to improve present estimation through state forecasting
  - Processing sequences of snapshots of SCADA data
  - Rate of processing: seconds

- **Static State Estimation including PMU Measurements**

  *Can be solved at SCADA rate or faster by parallelization and other improvements*

  *Interesting academic approaches but no practical interest*

  *Evolving technologies Transition stage*
Static State Estimation with PMU Data only

**PMU Data**
- Data: nodal voltage and branch current phasors
- Linear relationship with state variables (nodal voltage phasors)

**Measurement Model**

\[ z = B V + v \]
\[ v \sim (0, R) \]

\[
\begin{bmatrix}
Z_v \\
Z_I
\end{bmatrix} =
\begin{bmatrix}
I & 0 \\
Y_{IM} & Y_{IC}
\end{bmatrix}
\begin{bmatrix}
V_M \\
V_C
\end{bmatrix} +
\begin{bmatrix}
v_I
\end{bmatrix}
\]

**Estimator (Linear WLS)**

\[
\hat{V} = [B^H R^{-1} B]^{-1} B^H R^{-1} z
\]

**Requirements**
- PMUs installed in about 1/3 network nodes
- Adequate for local estimation
Static State Estimation with SCADA and PMU Data

- **Data**
  - Active and reactive power injections and flows, voltage magnitudes and **phase angles, branch currents**
  - Non-linear relationship with state variables (nodal voltage magnitudes and phase angles)

- **Measurement Model**
  \[ z = h(x) + v \]
  \[ v \sim (0, R) \]

- **Estimator (Non-linear WLS)**
  \[ x^{i+1} = x^i + [H(x^i)^T R^{-1} H(x^i)]^{-1} H(x^i)^T R^{-1} z \]

- **Difficulties**
  - Inclusion of current and phase angle measurements
  - Phase angle reference
  - Synchronization of SCADA and PMU data
  - **Software reprogramming**
Compromise Solution

State Estimation Combination

Conventional State Estimator

Linear State Estimator

Real Time Data Base
Static Data Base
PMU Data Base

SCADA

PDCs and Communication Network

RTU₁
PMU₁
RTU₁
PMU₁
RTU₁
PMU₁

Control Center
Hybrid State Estimation

- Also referred to as estimator with a priori information
- Combines the state estimation obtained a priori from a conventional state estimator with data from PMU
- Mathematical Model

\[
\begin{bmatrix}
\hat{V}_{SE} \\
Z_{PMU}
\end{bmatrix} = \begin{bmatrix} I \\ L \end{bmatrix} V + \begin{bmatrix} v_{SE} \\ v_{PMU} \end{bmatrix}
\]

\[v_{SE} \sim \begin{pmatrix} 0, H^T(\hat{V}_{SE}) \end{pmatrix} R^{-1} H(\hat{V}_{SE})\]

\[v_{PMU} \sim (0, Q_{PMU})\]
Hybrid State Estimation (cont.)

- There is no need for global observability of the PMU network
- Only observable state by the PMU network will be improved by the inclusion of this kind of information
- Useful to use PMU data to observe parts of the system (line, transformers, etc.) while keeping consistency with the SCADA estimator results
Fusion of Estimates

- Combines the state estimation obtained from the SCADA state estimator with the estimation obtained from the PMU linear estimator

Mathematical Model

- SCADA estimator results
  State estimation: $\hat{x}_{SCA}$
  Error covariance: $G_{SCA} = H^T(\hat{x}_{SCA}) R^{-1} H(\hat{x}_{SCA})$

- PMU linear estimator results
  State estimation: $\hat{x}_{PMU}$
  Error covariance: $G_{SCA} = B^H R^{-1} B$

- Fusion estimation
  $$\hat{x}^* = [G_{SCA} + G_{PMU}]^{-1} (G_{SCA}\hat{x}_{SCA} + G_{PMU}\hat{x}_{PMU})$$
Comparison

- Simulated experiments indicate that both the Hybrid State Estimation (HSE) and the Fusion of Estimates (FE) present an adequate performance in terms of the accuracy of estimates.

- The FE estimator requires the use of pseudo-measurements in the case of lack of observability in the PMU measurement system.

- Gross error detection and identification in the SCADA estimator should be performed as usual.

- The same procedures for gross error detection and identification can be applied to the HSE and FE estimators.
Dynamic State Estimation
Short Term Electromechanical Dynamics

- **History**
  - Few attempts in the past owing to the lack of adequate wide area data acquisition system
  - Some early works
    - Chang, Taranto, and Chow (1995)

- **PMU Availability Opened New Possibilities**
  - Smoothing: model identification
  - Filtering: better estimate of generator and controller internal variables
  - Prediction: security actions and protection
Kalman Filter

- Operates recursively on streams of noisy input data to produce a statistically optimal estimate of the underlying system state
- Two-step process
  - **Prediction Step**: produces estimates of the current state variables, along with their uncertainties
  - **Correction Step**: updated the estimate of the current state variables using a weighted average, with more weight being given to estimates with higher certainty

Kalman Filter (Linear Case)

Time Update ("Predict")

1. Project the state ahead
   \[ \hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1} \]
2. Project the error covariance ahead
   \[ P_k^- = AP_{k-1}A^T + Q \]

Measurement Update ("Correct")

1. Compute the Kalman gain
   \[ K_k = P_k^-H^T(HP_k^-H^T + R)^{-1} \]
2. Update estimate with measurement \( z_k \)
   \[ \hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-) \]
3. Update the error covariance
   \[ P_k = (I - K_kH)P_k^- \]

Initial estimates for \( \hat{x}_{k-1} \) and \( P_{k-1} \)

Source: Welch and Bishop
Extended Kalman Filter (EKF)

Time Update ("Predict")

1. Project the state ahead
   \[ \hat{x}^-_k = f(\hat{x}_{k-1}, u_{k-1}, 0) \]

2. Project the error covariance ahead
   \[ P^-_k = A_k P_{k-1} A_k^T + W_k Q_{k-1} W_k^T \]

Measurement Update ("Correct")

1. Compute the Kalman gain
   \[ K_k = P^-_k H_k^T (H_k P^-_k H_k^T + V_k R_k V_k^T)^{-1} \]

2. Update estimate with measurement \( z_k \)
   \[ \hat{x}_k = \hat{x}_k^- + K_k (z_k - h(\hat{x}_k^-, 0)) \]

3. Update the error covariance
   \[ P_k = (I - K_k H_k) P_k^- \]

Initial estimates for \( \hat{x}_{k-1} \) and \( P_{k-1} \)

Source: Welch and Bishop

\[ A_{k-1} = \left. \frac{\partial f}{\partial x} \right|_{\hat{x}_{k-1}} \]
\[ W_{k-1} = \left. \frac{\partial f}{\partial w} \right|_{\hat{x}_{k-1}} \]
\[ H_k = \left. \frac{\partial h}{\partial x} \right|_{\hat{x}_k^-} \]
\[ V_k = \left. \frac{\partial h}{\partial v} \right|_{\hat{x}_k^-} \]
Example: SMIB

- **Generator Model**
  - Classical model: constant voltage behind direct axis transient reactance

- **State Space Model**

  \[
  \begin{bmatrix}
  \delta_k \\
  \omega_k 
  \end{bmatrix} =
  \begin{bmatrix}
  \delta_{k-1} + (\omega_{k-1} - \omega_0) \Delta t \\
  \omega_{k-1} + \frac{\omega_0}{2H} \left( P_m - \frac{|E||V_{\infty}|}{x'_d + x_T + x_L} \sin \delta_{k-1} \right) \Delta t 
  \end{bmatrix} + [w_{k-1}]
  \]

- **Measurement Model**

  \[
  \begin{bmatrix}
  |V_k| \\
  \theta_k 
  \end{bmatrix} = h(\cdot) + v_k
  \]

  \[
  |V_k| \angle \theta_k = \frac{X_LE + X'_d V_{\infty}}{X_L + X'_d}
  \]

  \[
  E_k = |E_k| \angle \delta_k \\
  V_k = |V_k| \angle \theta_k \\
  V_{\infty} = |V_{\infty}| \angle 0^0
  \]
Example: SMIB (cont.)

- Kalman Filter Elements

\[ A = \frac{\partial g(\cdot)}{\partial x} = \begin{bmatrix} \frac{1}{2H} \left( \omega_0 \frac{|E||V_\infty|}{X'_d + X_T + X_L} \cos \delta \right) \Delta t & \Delta t \\ \end{bmatrix} \]

\[ H = \frac{\partial h(\cdot)}{\partial x} \]

\[ V = \frac{\partial h(\cdot)}{\partial v} \]
Extension to Multi-Machine Case

\[ x_k = \begin{bmatrix} \delta_1^k \\ \omega_1^k \\ \vdots \\ \delta_m^k \\ \omega_m^k \end{bmatrix} \]

\[ E_k = \begin{bmatrix} E_1^k \\ \vdots \\ E_m^k \end{bmatrix} \]

\[ I_{Gk} = \begin{bmatrix} I_{g1}^k \\ \vdots \\ I_{gm}^k \end{bmatrix} \]

\[ V_k = \begin{bmatrix} V_1^k \\ \vdots \\ V_m^k \end{bmatrix} \]

\[ I_{gk}^i = \frac{E_k^i - V_k^i}{jX_{di}} \]

\[ P_{ei} = \Re\{E_k^i I_{gk}^i*\} \]

\[ \begin{bmatrix} I_{Gk} \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GL} \\ Y_{LG} & Y_{LL} \end{bmatrix} \begin{bmatrix} E_k \\ V_k \end{bmatrix} \]

\[ V_k = -[Y_{LL}]^{-1}Y_{LG}E_k \]
DSE Comments

- More elaborated machine and controllers models have already been incorporated to DES with reasonable results
- Prediction Step can use electromechanical simulation techniques and software already available for simulation studies
- Parallel processing can speed up computations (Faster than Real Time)
- Allows the estimation of internal variables and parameters not accessible for measurements
- Use in wide area control requires relatively small PMU data delays
Final Remarks

- Synchrophasor information brought new ideas and problems to the state estimation area
- Conventional state estimation (static) may benefit from the accuracy and synchronization of PMU data but requires
  - Alteration in the available estimator code or
  - Methodology to combine conventional and PMU only estimates
- Dynamic state estimation may become a reality owing to the availability of high rates PMU data
  - Practical application areas are still not quite clear
  - Numerical difficulties with Kalman filter and derived algorithms have to be dealt with
  - Large computer requirements may be overcome by the use of very efficient electromechanical simulation software and parallel processing
Bibliography


5. B.L.D.C. Fonseca e D.M. Falcão, Implementação e Comparação de Resultados entre Estimadores de Estado Multiestágios, Submetido ao *XXIII SNPTEE*, outubro de 2015.


Thank You

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